Currency-hedging implementation issues

Adrian F Lee
JP Morgan Investment

This chapter addresses the following issues: the appropriate strategy adjustment for international investment managers whose normal currency exposure is hedged; the relative benefits of adding a currency overlay, comparing international fixed-income versus equity portfolios; and the transaction costs and cash flows associated with currency hedging.

In summary, we conclude that if a manager's performance is measured against a benchmark, we do not believe there is a logical justification for changing strategy simply because the currency benchmark or any part of the benchmark has altered. This is equally valid for international fixed-income and equities.

No special distinction should be made between international equity and fixed-income in identifying which portfolios to overlay or hedge individually. Conceptually, the problem should be considered as one of identifying the best group of managers to control aggregate currency exposure. To the extent these managers happen also to be existing international managers, then they will be required to overlay their own portfolios in addition to those of the remaining international managers. In practice, there is likely to be greater overlap on the fixed-income side than on the equity side.

Transaction costs for currency hedging have four sources: spot spreads, rollover spreads, market impact and cash flow related transaction costs. These appear to be small and, with active management, can be reduced.

Characteristics of multicurrency portfolio construction

Implications of a change in the benchmark and use of an overlay manager

(1) Multicurrency portfolio construction in absolute terms or versus an index should be thought of as the sum of two separate but not

independent portfolio problems — asset or country allocation and currency allocation. The former evaluated by comparing expected local returns *plus* the forward premium and the latter evaluated by comparing expected currency returns *minus* the forward premium. Asset/currency variances and covariances also require evaluation (see Appendix A).

(2) Total portfolio optimisation and relative portfolio optimisation trivially result in different portfolio structures. For a given level of total return, they can be related to one another as shown in Appendix B.

(3) Portfolio construction versus an index reduces to a problem of identifying *optimal deviations* from the asset and currency benchmark. For any given set of expectations and required excess return these optimal *deviations* do not depend on the benchmark weights. So a change of, say, currency and/or asset benchmark will not affect optimal asset or currency allocation when expressed in terms of deviations from the index (see Appendix C).

(4) If one manager is responsible for asset allocation and another for currency allocation, then the resulting set of manager bets may not be globally optimal. This is a standard problem associated with multiple managers and not attributable to any unique characteristics of multicurrency portfolio construction.

Appendices A, B and C we believe substantiate the conclusions above. Appendix A solves the total portfolio optimisation problem in a multicurrency context. Appendix B relates absolute and relative optimisations and Appendix C solves the relative optimisation problem in a multicurrency context.

The following text discusses these concepts and their practical implications for international investors.

Total portfolio return of a multicurrency portfolio is the sum of the asset weights times the local return *plus* the forward premium added to the currency weights times the currency return *minus* the forward premium — see Equation 1 of Appendix A. This is useful because it tells us that, in determining asset or country exposure, a portfolio manager should compare local returns *adjusted* by the forward premium — ie, what he gets for a pure asset bet separately from currency. An equivalent statement for currencies can be made. An illustration of these comparisons is given in Table 1.

Table 1: Total return of a multicurrency portfolio

	Forecast returns					Deviations over/under weights	
	Local	Curr	Forward prem	Local + FP	Curr — FP	Assets	Curr
US	16	0	0	16	0	10	5
Japan	7	3	4	11	—1	—30	0
Germany	15	5	3	18	2	10	5
UK	20	—5	—2	18	—3	10	—10

A key practical issue facing sponsors in the multicurrency context is whether the presence of a currency overlay manager impacts the optimal strategy of the underlying manager, such that the latter should alter his asset bets from what they would otherwise be? For example, if, in the absence of an overlay manager the asset manager overweights German bonds versus UK gilts, then could this strategy be inappropriate (given no change in expectations) when an overlay manager simultaneously hedges the currency exposure?

The answer to the question is "no" to the extent that the overlay manager simply alters the currency *benchmark*, but "possibly" to the extent that the overlay manager changes the original manager's *currency bets*. It is important not to confuse these two different cases.

In the first case, an active manager's objective is to maximise excess return over an index and minimise volatility of this excess return. In this context, a manager's decisions are expressed in the form of over and under-weightings versus an index. The optimal solutions for these deviations do not depend on the index that they deviate from. Equation 9 in Appendix C shows that these deviations (for a given level of excess return) depend on the expected local returns, currency returns, forward premium and asset currency variance/covariance matrix — not in any way on the actual asset or currency index. See Tables 2 and 3 for an illustration of this.

An intuitive way of understanding this is to observe that residual risk and return are direct functions of manager *deviations* from benchmark — that actual benchmark risk and covariance are subtracted out of portfolio risk and return in the evaluation of the manager's portfolio. Therefore, to the extent that an overall benchmark currency exposure is altered by an overlay manager, then the underlying manager should not alter his asset or currency bets; a useful analogy is given in the next section.

Table 2: Portfolio construction versus unhedged index

	Index		Optimal portfolio		Implied
	Asset	Currency	Asset	Currency	hedge
US	50	50	60	55	5
Japan	30	30	0	30	(30)
Germany	10	10	20	15	5
UK	10	10	20	0	20

Table 3: Portfolio construction versus 50% hedged index

	Index		Optimal portfolio		Implied
	Asset	Currency	Asset	Currency	hedge
US	50	75	60	80	(20)
Japan	30	15	0	15	(15)
Germany	10	5	20	5	15
UK	10	5	20	0	20

With respect to the second case, if the overlay manager alters the currency bets of the original manager, then the overall bet structure may not be optimal. This problem is the standard multiple managers' situation where the aggregate of two different managers' bets may not be optimal from the overall portfolio viewpoint. A familiar example of this problem is where separate bond and equity managers both expect interest rates to decline and both bet on this factor. If the equity manager knew of the bond manager's decisions, he would not necessarily duplicate this bet if he wished to manage overall portfolio risk and return.

Role of a currency overlay manager

International equity versus fixed-income

An active currency overlay manager does two things:

- changes the normal currency exposure of a portfolio; and
- alters the active currency bets of the underlying manager.

In general, the benefits of using a specialised currency overlay manager, versus letting each fund manager hedge, are outlined below.

No disruption of individual managers

Individual managers' roles, strategy or performance measurement, should not alter if currency exposure is hedged. We do not believe their asset bets should logically be altered in the presence of tactical or longer-term hedging. We discussed this above. Obviously, if managers state that they would change asset strategy in the presence of an overlay manager, this behavioural tendency must be examined. Presumably they would also change these asset and currency bets if they did the hedging themselves. This tendency suggests that managers may not be actually thinking about *residual* return and risk but more in terms of total risk. It is not clear that this is desirable from the sponsor's point of view.

A helpful analogy for approaching this problem is to think of the individual managers as balanced domestic managers investing in equities and bonds where the stock/bond percentage is fixed (asset and currency exposure must always be 100% each), and the overlay manager as altering the bond duration strategically from the balanced manager's benchmark and also adding duration bets around the new index. This analogy is also appropriate in that stocks and bonds are about as related as assets and their own currencies. The argument that the balanced manager should become either more defensive or aggressive in his equity portfolio, because the overlay manager has, say, reduced his bond duration, ignores the fact that his stock selection is still measured in the same way and that his expectation should still be incorporated in the same way in equity bets and bond bets. A strong correlation between equity and bond returns does not change this. The overall portfolio may be sub-optimal but this is the standard multiple manager's problem.

The difference between the balanced manager analogy and the currency overlay situation is that, in the balanced situation there would be no need for a bond manager (unless they had some other specialist bond skill or insight). In this event the assets would be transferred to the overlay manager for management and the overall fees reduced. In the international portfolio case one automatically gets a currency management with asset management and these portfolios cannot be transferred and can only be separately managed through overlay procedures.

Use of specialist currency manager skills

The critical question in deciding on the use a currency specialist, irrespective of if they are also altering the normal currency exposure, is whether or not the use of the specialist increases the currency information ratio of the *total* currency portfolio — ie, the *expected excess* return to excess risk ratio, attributable to active currency management.

In this context two parameters are important:

- the excess return/risk ratio of each manager; and
- the correlation of excess returns across managers.

In general, managers who have high information ratios which are uncorrelated should be used to control the currency exposure of the portfolio. These managers can be individual and/or overlay managers.

The key concept is to view the total currency portfolio as one, and in isolation from the assets, and then to put in place a structure of currency managers that maximises the aggregate currency information ratio. These currency managers could also be asset managers but asset and currency responsibilities should be viewed as separate. There is no reason that a manger should control the same or an equal size of asset and currency exposure, unless their information ratio on each was identical when viewed against all other managers. An illustration may be helpful here.

Figure 1: Information ratios and international investments — an example

Traditional structure

	Bonds		Equities			
Manager	A	B	C	D	E	Aggregate
Assets	1:2	1:3	1:2	1:3	1:3	1:1.2
Currencies	1:2	1:3	1:4	1:5	1:6	1:1.9

New structure

	Bonds		Equities			
Manager	A	B	C	D	E	Aggregate
Assets	1:2	1:3	1:2	1:3	1:3	1:1.2

	A	B	C	
Currencies	1:2	1:3	1:4	1:1.7

In the example of Figure 1, the traditional structure has five managers with expected excess return/risk ratios as illustrated in the boxes — these have been separately identified for assets and currencies. Each manager controls an equal amount of assets and currencies, and excess returns by managers are assumed independent in the calculation of aggregates, although this assumption is not critical.

In the new structure managers A, B and C are given equal control of all currency exposure and the aggregate information ratio is raised from 1:1.9 to 1:1.7. This is a result of their higher individual information ratios and diversification among them. In practice, this would mean that each controls the currency associated with the assets they invest plus overlay one third of the currency exposure associated with D and E.

Easier overall currency exposure and reduced overall cash flow related transaction costs

An overlay approach will imply significantly less cash flow related transaction costs. Individual managers will use underlying international assets to fund and reinvest hedging losses or gains. This will imply higher transaction costs to the extent international equities or bonds are more expensive to transact than domestic assets. An overlay manager coordinates hedging cash flows with the general cash of the investor. This can reduce transaction costs from 60bp per annum to 25bp.

Using an overlay manager(s) will more likely facilitate the plan sponsor's overall control of currency exposure on an ongoing basis. The normal hedge position can be readily changed without having to alter individual manager benchmark and performance measurement techniques.

Cash flow requirements depend on currency volatility, length of hedges and the degree of active management. The results of simulation analysis of these variables are discussed in the next section.

Transaction costs associated with currency hedging

There are four sources of transactions associated with hedging currency exposure:

- spread of the forward exchange contracts;
- spread on the rollover of forward exchange;
- market impact; and

- transaction costs associated with investing or funding cash flows generated by the hedge portfolio.

Forward spread

This is the difference between the bid and ask rates for buying or selling currency forward at a specific horizon. This spread is paid on the initial amount hedged and on any new cash flow or exposures to be hedged. It is the sum of the spot spread and the spread on the forward points. Table 4 shows these spreads one month forward for Swiss francs, Deutsche marks and yen. For the Swiss franc it is 4.9bp.

Rollover spread

This is the spread on the forward points paid for hedging. It is the spread paid per month to roll over the forward position. There is no spot spread paid at this point as the investor has already taken the short or long position and is simply rolling it over. For example, the rollover spread on DM month is 1.2bp or 13.9bp per annum.

To hedge for a year, the investor pays one forward spread and 11 rollover spreads, so the annualised rollover spread is the appropriate measure of annual transaction costs. In our example, this is 13.9bp for the DM and 9.5bp for the yen using a monthly hedge rolled over. The equivalent costs for longer-term six-month hedging are less, as shown in Table 4.

Market impact

Market impact of hedging forward exchange is difficult to estimate. This impact is a function of market liquidity and the volume of the transaction at any given point in time.

Daily volume of the interbank foreign exchange markets is estimated at over $100–200bn per day. In smaller currencies and at certain times of the year, this liquidity is significantly less, for example, at holiday periods. The most liquid market is generally for hedging into the US dollar, as opposed to cross-hedging into other currencies.

The spreads quoted in Table 4 are for a "normal" transaction size of $5–10m. For greater or lesser amounts a specific quote is necessary, either specifying which side the investor is on or leaving this open. Spreads may or may not be higher for these larger or smaller amounts.

We believe that the transaction costs in Table 4 reflect the costs realistic for larger institutional portfolios given active trading. Market impact should also be minimal with active trading.

Table 4: Rollover cost of an FX position

	Spot market		Forward points		Forward rate	
Currency	$/SwFr	SBBX	1	month	(discount)	
Spread	5	(pips)	2	(pips)		
Bank buys $	1.4425		58		1.4367	
Bank sells $	1.4430		56		1.4374	
Basis points cost	3.5		1.4		4.9	
			16.6	pa	58.2	pa
Currency	$/DM	SBBX	1	month	(discount)	
Spread	10	(pips)	2	(pips)		
Bank buys $	1.7235		60		1.7175	
Bank sells $	1.7245		58		1.7187	
Basis points cost	5.8		1.2		7.0	
			13.9	pa	83.6	pa
Currency	$/yen	BOTX	1	month	(discount)	
Spread	5	(pips)	1	(pips)		
Bank buys $	126.30		37		125.93	
Bank sells $	126.35		36		125.99	
Basis points cost	4.0		0.8		4.8	
			9.5	pa	57.0	pa
Currency	$/SwFr	SBBX	6	month	(discount)	
Spread	5	(pips)	5	(pips)		
Bank buys $	1.4425		350		1.4075	
Bank sells $	1.4430		345		1.4085	
Basis points cost	3.5		3.5		6.9	
			6.9	pa	13.9	pa
Currency	$/DM	SBBX	6	month	(discount)	
Spread	10	(pips)	5	(pips)		
Bank buys $	1.7235		343		1.6892	
Bank sells $	1.7245		338		1.6907	
Basis points cost	5.8		2.9		8.7	
			5.8	pa	17.4	pa
Currency	$/yen	BOTX	6	month	(discount)	
Spread	5	(pips)	2	(pips)		
Bank buys $	126.30		216		124.14	
Bank sells $	126.35		214		124.21	
Basis points cost	4.0		1.6		5.5	
			3.2	pa	11.1	pa

The spot rollover should be done without spread. The quoting bank is taking a spread on the forward points only.

Cash flow related costs

Aggregate currency gains or losses in an overlay portfolio or individual manager portfolio must be ultimately reinvested or funded. There will be transaction costs associated with these cash flows. These cash flows will be less if they are aggregated into an overlay portfolio.

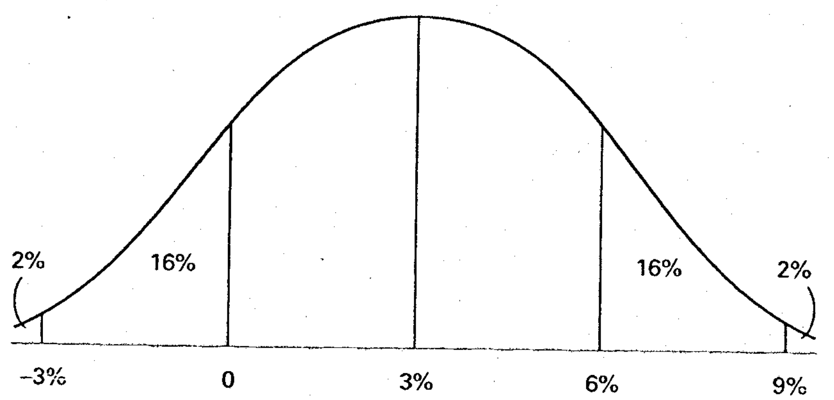
The question here is how frequently assets must be rebalanced and what level of cash balances should be tolerated. The following analysis is illustrative of the sort of transaction costs that may be expected.

The historical standard deviation of monthly aggregate currency gains or losses has been about 3% monthly or 5% quarterly. Given that outstanding losses and gains can be calculated daily but only become due, say, at the end of the month, then cash balances can be kept to a minimum when planning for losses.

If one assumes that the average cash balance held to fund losses was 3% of the amount hedged, then, assuming a normal distribution of currency returns, the monthly distribution of cash balances would look as shown in Figure 3.

If one assumes that 3% should be transferred out whenever cash balances reach 6%, then there is a 16% probability of this occurring or a frequency of about two out of 12 months. Likewise, two out of 12 months at 3% inflow will be required. This implies a total volume of transactions of 12% per annum. Assuming one-way transaction costs of 0.50%, then total cost to the portfolio is 6bp per annum. The transaction cost figure used here should not necessarily be exclusively an international one as these cash flows will be reinvested or funded from the total portfolio in the absence of any specific overall asset allocation strategy.

Figure 2: Monthly distribution of cash balances



Cash flows associated with currency hedging

Simulation techniques we used attempt to address several questions:

- What impact does currency volatility have on cash flow requirements? How big a problem is serial correlation of currency return, ie, trends?
- What is the effect of layered hedges, eg, monthly and quarterly hedging combined?
- What impact does the active management of currency exposure have on the whole process?
- What are the cash flow requirements in a multiple manager environment?

The results of these are given in Appendix D. The model assumed non-stochastic, ie, fixed forward premia, no rebalancing, a 100% hedged normal position and simulated 200 currency returns each month. The conclusions of this analysis are:

(1) the standard deviation of monthly cash flow is likely to be about 3% per month;

(2) serial correlation of currency return could increase the annual volatility from 10.4% to 14% or 15%;

(3) layered hedging, ie, using combinations of monthly, quarterly and one-year forward contracts significantly reduces monthly cash flow volatility;

(4) active management on average reduces cash flow volatility, because aggregate hedging levels are reduced — ie, active management on average reduces hedging, given a fully-hedged benchmark;

(5) active management insight reduces slightly cash flow volatility and generates a positive net cash flow on average;

(6) multiple overlay managers reduce *net* cash flow requirements but add to absolute cash flow, ie, implies between overlay manager cash flows. This intra-manager cash flow is less volatile if there is a higher correlation between the multiple managers.

Appendix A: Multicurrency portfolio optimisation — investor's problem

Total portfolio return:

$$= \Sigma(w' - h')([(1 + r')(1 + c')] - 1) + h'(r' + FP')$$

$$= \Sigma(w' - h')(r' + c') + h'(r' + FP') \text{ assuming } r' \cdot c' = 0$$

$$= \Sigma w'(r' + FP') + \Sigma(w' - h')(c' - FP') \text{ [subtract and add } \Sigma w'FP'\text{]}$$

$$= \Sigma \text{ asset weight } (r' + FP') + \Sigma \text{ currency weight } (c' - FP)$$

where:
w' = asset or country weight
h' = hedge weight
r = local asset return
c = currency return
FP = forward premium versus base currency
x' = w' — h' currency exposure

Equation 1: Expected return

$$E(PR) = \Sigma w' E(r' + FP') + \Sigma x' E(c' - FP)$$

$$= \Sigma w'(E(r') + FP') + \Sigma x'(E(c') - FP')$$

Equation 2: Converting to matrix notation

Let:

$\begin{bmatrix} W \\ X \end{bmatrix}$ = a 2 n vector of asset and currency weights

$\begin{bmatrix} R \\ C \end{bmatrix}$ = a 2 n vector of local return and currency return adjusted by the forward premium

1 = a unitary vector of length n

Investor's problem is:

$$\text{Min Var} \begin{bmatrix} W \\ X \end{bmatrix}' \begin{bmatrix} R \\ C \end{bmatrix} \quad \text{ST} \begin{bmatrix} W \\ X \end{bmatrix}' \; E \begin{bmatrix} R \\ C \end{bmatrix} = \mu$$

$$\text{ST} \begin{bmatrix} W \\ X \end{bmatrix}' \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = [1.0 \; 1.0]$$

This can be rewritten as follows:

$$\text{Min: Var}(Z'f) \;\; \text{ST} \;\; Z' E(f) = \mu$$

$$Z' \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = (1.0, 1.0)$$

or: ST $Z' M = K$

where:

$M = \begin{pmatrix} E(f) & 1 & 0 \\ & 0 & 1 \end{pmatrix}$, $\quad k = (\mu, 1.0, 1.0)$ and $f = \begin{bmatrix} R \\ C \end{bmatrix}$

The Lagrangean in Z and λ is:

Equation 3

$$L_{z,\lambda} : Z'\Sigma Z - Z'(M - K)\lambda,$$

Σ is variance covariance matrix of R, C and λ is vector are Lagrangean multipliers.

The solution to this problem is:

$$Z^* = K(M'\Sigma^{-1} M)^{-1} M'\Sigma^{-1}$$

Resubstituting:

Equation 4

$$\begin{pmatrix} W^* \\ X^* \end{pmatrix} = (\mu, 1.0, 1.0) \left[\begin{pmatrix} E(f) & 1 & 0 \\ & 0 & 1 \end{pmatrix}' \Sigma^{-1} \begin{pmatrix} E(f) & 1 & 0 \\ & 0 & 1 \end{pmatrix} \right]^{-1} \; E(f) \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}' \Sigma^{-1}$$

Appendix B: Relationship between total return and excess return optimisation

Quadratic optimisation problem

X = vector of portfolio weights	$(n \times 1)$
μ = vector of expected asset returns	$(n \times 1)$
V = variance covariance matrix	$(n \times n)$
C = constant	scalar
1 = unitary vector	$(n \times 1)$
$\lambda_1 \lambda_2$ = vector of Lagrangean cost multipliers	

$$\text{Min } X'VX \text{ ST. } X[\mu \;\; 1] = (C \;\; 1.0) : A$$

$$L_X : X'VX - (X'[\mu \;\; 1] - [C \; 1.0]) \begin{bmatrix} \lambda_1 \\ \lambda_2 \end{bmatrix}$$

Let:

$[\mu \;\; 1] = \mu o$ n x 2 matrix

$[C \; 1.0] = Co$ 1 x 2 matrix

$\begin{bmatrix} \lambda_1 \\ \lambda_2 \end{bmatrix} = \lambda o$ 2 x 1 matrix

$$L_X = X'VX - [X'\mu o - Co]\lambda o$$

Equation 5

$$\frac{\delta Lx}{X} = 0 \quad \Rightarrow \quad 2X^{*\prime}V - (\mu o \lambda^{*} o)' = 0$$

Equation 6

$$\frac{\delta Lx}{\delta \lambda o} = 0 \quad \Rightarrow \quad X^{*\prime}\mu o = Co$$

from Equation 5 $X^* = \dfrac{(\mu o \lambda^* o)' V^{-1}}{2} = \dfrac{\lambda^{*\prime} o}{2} \mu' o V^{-1}$

from Equation 6 $X^* \mu o = Co = \dfrac{\lambda^{*\prime} o}{2} \mu' o V^{-1} \mu o$

Therefore: $\dfrac{\lambda^{*\prime} o}{2} = Co(\mu' o V^{-1} \mu o)^{-1}$

Substituting for $\dfrac{\lambda^{*\prime} o}{2}$

Equation 7

$$X^{*\prime} = Co(\mu' o V^{-1} \mu o)^{-1} \mu' o V^{-1}$$

Excess return optimisation problem

Min : $Var(X'\mu - W'\mu)$	ST	$X'\mu - W'\mu = K$	: B
		$(X' - W')1 = O$	
: $Var\ Z'\mu$	ST	$Z'\mu = K$	: C
		$Z'1 = O$, where $Z = X - W$	
		1 is a unitary vector	

From the Lagrangean: $L_z : Z'VZ - Z'[\mu o - Ko]\ Lo$

where:

$Lo = \begin{bmatrix} L_1 \\ L_2 \end{bmatrix}$

vector of Lagrangean constraints

$Ko = (K,O)$

From Equation 7 the optimal solution is given by:

$Z^* = Ko(\mu o' V^{-1} \mu o)^{-1} \mu' o V^{-1}$

To relate X* and Z*, let $C = w'\mu + K$

then:

$$Z^* = [C - W'\mu, O]\,(\mu o' V^{-1} \mu o)^{-1} \mu o' V^{-1}$$
$$= [C - W'\mu, O]\,\pi$$

where:

$$\pi = (\mu o' V^{-1} \mu o)^{-1} \mu o' V^{-1}$$

From Equation 7, $X^* = [C, 1.0]\,\pi$

Subtracting:

$$X^* - Z^* = (W'\mu, 1.0)\,\pi$$
$$= (W'(\mu\ 1)\,\pi \text{ as } W'1 = 1.0$$
$$= W'\,\mu o\,\pi$$
$$= W'\,\mu o(\mu o' V^{-1} \mu o)^{-1} \mu o' V^{-1}$$
$$= W'P \text{ where P is an idempotent matrix with characteristic roots} = 1$$

Appendix C: Multicurrency portfolio excess return optimisation — manager's problem

From Appendix A, Equation 2

Portfolio return (PR) $= \begin{bmatrix} W \\ X \end{bmatrix}' \begin{matrix} R \\ C \end{matrix}$ Index return (IR) $= \begin{bmatrix} \overline{W} \\ \overline{X} \end{bmatrix}' \begin{bmatrix} R \\ C \end{bmatrix}$

Excess return $= PR - IR = \begin{bmatrix} W - \overline{W} \\ X - \overline{X} \end{bmatrix}' \begin{bmatrix} R \\ C \end{bmatrix} = \begin{bmatrix} d_w \\ d_x \end{bmatrix}' \begin{bmatrix} R \\ C \end{bmatrix}$

Equation 8

Manager's problem is Min Var $\begin{bmatrix} d_w \\ d_x \end{bmatrix}' \begin{bmatrix} R \\ C \end{bmatrix}$ ST $\begin{bmatrix} d_w \\ d_x \end{bmatrix}' E \begin{bmatrix} R \\ C \end{bmatrix} = \epsilon$

$$\begin{bmatrix} d_w \\ d_x \end{bmatrix}' \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = [0\ 0]$$

where:
$\overline{W}$ = index asset weights
$\overline{X}$ = index currency weights
d_w = vector of asset over/under weights
d_x = vector of currency over/under weights
ϵ = required excess return

This can be rewrittten as Min Var (D'f) ST D'E(f) $= \epsilon$

$$D' \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = (0.0)$$

or: D'N = j

Forming the Lagrangean:

L_{D}, I : D'SD — D'(N —j)I

where:

$$N = \begin{bmatrix} E(f) & 1 & 0 \\ & 0 & 1 \end{bmatrix}$$

$j = (\epsilon,0,0)$

Solving the Lagrangean in D and λ gives

Equation 9

$$D^* = j\,(N'S^{-1}N)^{-1}\,N'S^{-1}$$

or:

$$\begin{pmatrix} d_w \\ d_x \end{pmatrix}' = (\epsilon,0,0) \left[\begin{pmatrix} E(f) & 1 & 0 \\ & 0 & 1 \end{pmatrix}' \Sigma^{-1} \begin{pmatrix} E(f) & 1 & 0 \\ & 0 & 1 \end{pmatrix} \right]^{-1} \begin{pmatrix} E(f) & 1 & 0 \\ & 0 & 1 \end{pmatrix}' \Sigma^{-1}$$

For an unhedged index $X = W$
For a fully-hedged index $X = O$

Therefore optimal deviations are independent of normal position for given excess return.

Appendix D: Currency overlay cash flow simulation

Table 5: Single currency, single manager, fixed single horizon

Cash flow volatility						
100% hedge, 3% monthly currency volatility						
Serial correlation:	**0**	**0.02**	**0.3**	**0.4**	**0.5**	**—0.1**
Monthly	3%	3.1%	3.2%	3.3%	3.4%	2.9%
Quarterly	5.2%	6.2%	6.5%	7.5%	7.7%	4.2%
Semi-annual	7.4%	9.3%	9.7%	11.5%	12.3%	5.5%
Annual	10.4%	12.2%	14.2%	17.5%	18.3%	7.6%
100% hedge, 6% monthly currency volatility						
Serial correlation	**0**	**0.2**	**0.3**	**0.4**	**0.5**	**—0.1**
Monthly	6%	6.1%	6.2%	6.5%	7.0%	5.9%
Quarterly	10.4%	11.6%	12.6%	14.0%	15.7%	8.7%
Semi-annual	14.7%	17.2%	18.7%	22.7%	24.5%	12.2%
Annual	20.8%	24.8%	26.3%	34.2%	37.0%	14.9%

Table 6: Single currency, single manager, fixed layered hedging

Currency volatility 3%	Cash flow volatility (%)											
Month Layer (MQSA)	1	2	3	4	5	6	7	8	9	10	11	12
1/2,1/2,0,0	1.4	1.5	3.3	1.4	1.5	3.5	1.4	1.4	3.4	1.5	1.5	4.0
1/3,1/3,1/3,0	0.9	1.0	2.4	1.0	1.0	4.6	1.0	0.9	2.2	0.9	0.9	4.2
1/4,1/4,1/4,1/4	0.7	0.7	1.7	0.7	0.7	3.0	0.7	0.7	1.8	0.7	0.7	5.7

Currency volatility 6%	Cash flow volatility (%)											
Month Layer (MQSA)	1	2	3	4	5	6	7	8	9	10	11	12
1/2,1/2,0,0	2.8	3.1	7.1	2.9	3.0	7.0	2.8	3.3	6.7	3.2	3.1	7.2
1/3,1/3,1/3,0	1.9	1.8	4.4	2.0	2.2	9.1	1.9	2.0	4.6	1.9	2.0	8.8
1/4,1/4,1/4,1/4	1.5	1.4	3.6	1.4	1.5	6.5	1.4	1.5	3.8	1.5	1.4	10.5

Table 7a: Single currency, one manager, active hedge, monthly horizon

Currency volatility 3%, forecast error volatility 3%, $B = 0$

	Min hedge			
	25%	50%	75%	100%
Monthly				
Net cash flow (%)	0	0	0	0
Cash flow volatility (%)	2.0	2.2	2.6	3.0
Average hedge	0.65	0.76	0.89	1.0
Quarterly				
Net cash flow (%)	0	0	0	0
Cash flow volatility (%)	3.4	4.0	4.6	5.2
Annual				
Net cash flow (%)	0	0	0	0
Cash flow volatility (%)	7.3	8.4	8.5	9.9

Table 7b: Single currency, one manager, active hedge monthly horizon
Minimum hedge 50%

	Forecast volatility 3%		B	Forecast volatility 1%	
	0	0.1	0.5	0.2	0.5
Monthly					
Net cash flow (%)	0	0.0	0.2	0.3	0.6
Cash flow volatility (%)	2.2	2.3	2.2	2.2	2.6
Average hedge	0.76	0.76	7.6	0.75	0.76
Quarterly					
Net cash flow (%)	0	0	0.7	1.1	1.6
Cash flow volatility (%)	4.0	4.0	3.9	3.9	3.7
Annual					
Net cash flow (%)	0	0.5	2.7	4.6	6.6
Cash flow volatility (%)	8.4	8.3	7.6	7.8	6.6

Table 8a: Single currency, multiple managers, active hedging monthly

Currency volatility 3%, B : 0, forecast volatility 3%, minimum hedge 50%, zero correlation between manager bets

	Number of managers					
	1		2		3	
	Mean	SD	Mean	SD	Mean	SD
Monthly						
Negative cash flow (%)	—0.8	1.3	—0.8	0.9	—0.8	0.7
Positive cash flow (%)	0.8	1.3	0.8	0.9	0.8	0.7
Net cash flow (%)	0	2.2	0	1.6	0	1.4
Absolute cash flow (%)	0	2.2	1.0	1.6	1.5	0.8
Quarterly						
Negative cash flow (%)	—2.6	2.4	—2.6	1.7	—2.5	1.4
Positive cash flow (%)	2.6	2.4	2.6	1.7	2.5	1.4
Net cash flow (%)	0	4.0	0	2.7	0	2.3
Absolute cash flow (%)	0	4.0	5.3	1.6	5.0	1.5
Annual						
Negative cash flow (%)	—10	5.1	—10.3	3.3	—10.4	2.8
Positive cash flow (%)	10	5.1	10.3	3.3	10.4	2.8
Net cash flow (%)	0	8.4	0	5.2	0	4.4
Absolute cash flow (%)	0	8.4	20.8	3.4	21.0	2.8

Table 8b: Single currency, two managers, active hedging monthly

Currency volatility 3%, B : 0, forecast volatility 3%, minimum hedge 50%

	Correlation					
	0		0.5		0.75	
	Mean	SD	Mean	SD	Mean	SD
Monthly						
Negative cash flow (%)	—0.8	0.9	—0.8	1.4	—0.8	1.5
Positive cash flow (%)	0.8	0.9	0.8	1.4	0.8	1.4
Net cash flow (%)	0	1.6	0	2.4	0	2.4
Absolute cash flow (%)	1.0	1.6	1.8	1.7	1.8	1.7
Quarterly						
Negative cash flow (%)	—2.6	1.7	—2.6	2.6	—2.6	2.7
Positive cash flow (%)	2.6	1.7	2.6	2.6	2.6	2.7
Net cash flow (%)	0	2.7	0	4.2	0	4.3
Absolute cash flow (%)	5.3	1.6	5.5	2.8	5.2	2.8
Annual						
Negative cash flow (%)	—10.3	3.3	—11.0	5.4	—10.9	5.5
Positive cash flow (%)	10.3	3.3	11.0	5.4	10.9	5.5
Net cash flow (%)	0	5.2	0	8.2	0	8.5
Absolute cash flow (%)	20.8	3.4	21.3	5.5	20.7	5.6

Figure 3: Trailing three-year currency volatility

Serial correlation 1983–88, based on quarterly returns